MEXT Monbukagakusho

EPS 01 MATHEMATICS | Coordinate Geometry

1. Given a graph of the following cubical function:
$$y = [\dots]x^3 + [\dots]x^2 + [\dots]x + [\dots]$$
is passing through $(0,1)$, $(-1,-7)$, $(1,7)$, and $(-2,-29)$.

2. In the domain of $x \in [1,4]$, a function $f(x)$
$$f(x) = |x| + |x^2 - 3x|$$
has the minimum value of [...] and the maximum value of [...].

3. An ellipse $4x^2 + y^2 = 4$ has a minimum y value of [...] and a maximum y value of [...].

4. Points $(-5,4)$ and $(3,-2)$ lie on a straight line. The shortest distance from point A$(-2,8)$ to the straight line is [...].

5. Given that a circle is centered at $(2,3)$ with radius of 5. The shortest distance between a point on the circle and the straight line $4x + 3y = 92$ is [...].
6. In the domain of $x \in [2,10]$, a function $f(x)$
$$f(x) = x^2 - 10x + 6$$
has a minimum value at $x =$ [...] and a maximum value at $x =$ [...].

7. Given two circles pass through two points $(3,1)$ and $(4,2)$ and are tangent to the x –axis. The radius of the smaller circle is [...], while the radius of the larger circle is [...].

8. Given that $f(x)$ is a linear function. If

$$\int_{\frac{k}{2}}^{2k} f(x)\,dx = -\frac{15}{8}(k^2 - 4k)$$

for $k > 0$, then $f(x) = \frac{[\ldots]\,x + [\ldots]}{2}$

9. Two circles, namely circle P with radius of 2 cm and circle Q with radius of r, intersect at two points on a plain. If a tangent line of circle P and a tangent line of circle Q are drawn at a particular intersecting point of those two circles, they form an angle of $120°$ outside of circle P and circle Q.
   a. Express the distance l between the center of P and Q in terms of r
   b. Find the value of r that gives value of l in (a) minimum
   c. For the value of r in (b), express the exact area of the intersection between circle P and Q in terms of π

a.	b.	c.

10. Given a function $y = 27^x - 9^{x+0.5} + 4$ for $x \in \mathbb{R}$.
   a. Let $= 3^x$. Express y in terms of A.
   b. Find the values of y at which the local minimum and local maximum occur, and the values of A in (a) at which y attains them.
   c. Find the coordinates of global maximum and global minimum of the function in the interval $0 \le x \le \log_3 4$.

a.	b.	c.

11. If a function $g(x)$ satisfies the following equation: $\int_c^x g(t)\,dt = 2x^2 + (c-1)x - 10$, then the constant c is [...] and the function $g(x) =$ [...]. The minimum value of the integral $\int_c^x g(t)\,dt$ is [...].

12. Given a function $f(x) = x^3 + ax + b$ where a and b are real constants.

a. If the function $f(x)$ is tangent to x-axis at $x = -2$, then find the values of a and b

b. By using the values of a and b obtained from (a), find any other value of x at which the function intersects with the x-axis.

c. By using the values of a and b obtained from (a), find the area of a region bounded by the function $f(x)$ and the x-axis.

a.	b.	c.

13. Solve the following equation $3a^2 - a + 2 = 0$ and express them in the form of $x + yi$ where x and y are real numbers.

$a_1 =$	$a_2 =$

14. If $\alpha = \frac{-3-3\sqrt{3}i}{2}$, then $\alpha^3 = [\ldots] + [\ldots]i$, where $i = \sqrt{-1}$ and the answers are real numbers.
15. If $\omega = 3\sqrt{2} - 3\sqrt{2}i$, then ω can be expressed as $r.e^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. $r = [\ldots]$ and $\theta = [\ldots]$.
16. Given complex numbers $a = 1 - 3i$ and $b = 3 + 2i$. Express $\frac{a}{b}$ in the form of $x + yi$ where x and y are real. $x = [\ldots], y = [\ldots]$
17. Express $\frac{i^3}{(\sqrt{3}-i)^2}$ in the form of $r.e^{i\theta}$ and $x + yi$ where x and y are real numbers and $-\pi < \theta \leq \pi$.

$r =$	$\theta =$	$x =$	$y =$

18. Given that $z_1 = 4\ cis\left(\frac{\pi}{6}\right)$ and $z_2 = 3\ cis\ (-\frac{\pi}{3})$. If $\beta = z_1^3 - z_2{}^*$ in which z_2^* is the conjugate of z_2, then express β in the form of $r.e^{i\theta}$ where $r > 0$ and $< -\pi < \theta \leq \pi$.

$\beta =$

Hukum I Termodinamika

$Q = \Delta U + W$

Q		ΔU		W	
$+$	Sistem menerima kalor	$+$	Suhu sistem naik	$+$	Sistem melakukan kerja (memuai)
$-$	Sistem melepas kalor	$-$	Suhu sistem turun	$-$	Sistem dikenai kerja (memampat)

Proses-proses Termodinamika

$\Delta T = T_2 - T_1$

$\Delta V = V_2 - V_1$

$c_p = R + c_v$

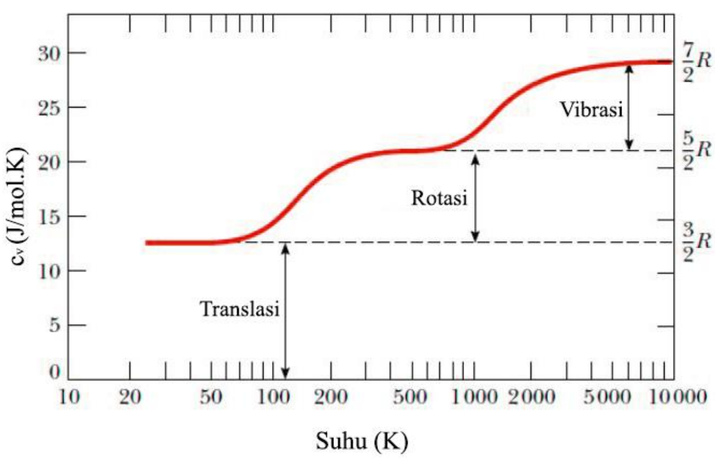
$\gamma = \frac{c_p}{c_v}$

Pada gas monoatomik: $c_v = \frac{3}{2}R, \quad \gamma = \frac{5}{3}$

Pada gas diatomik suhu rendah (lihat gambar): $c_v = \frac{3}{2}R, \quad \gamma = \frac{5}{3}$

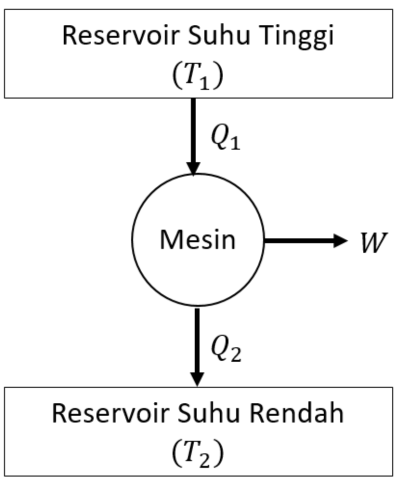
Pada gas diatomik suhu sedang (lihat gambar): $c_v = \frac{5}{2}R, \quad \gamma = \frac{7}{5}$

Pada gas diatomik suhu tinggi (lihat gambar): $c_v = \frac{7}{2}R, \quad \gamma = \frac{9}{7}$



Isobarik $P_1 = P_2$ $\frac{V_1}{T_1} = \frac{V_2}{T_2}$	$Q = n\, c_p\, \Delta T$	$\Delta U = n\, c_v\, \Delta T$	$W = P\, \Delta V$ $W = n\, R\, \Delta T$
Isokhorik $V_1 = V_2$ $\frac{P_1}{T_1} = \frac{P_2}{T_2}$	$Q = U = n\, c_v\, \Delta T$	$\Delta U = n\, c_v\, \Delta T$	$W = 0$
Isotermik $T_1 = T_2$ $P_1V_1 = P_2V_2$	$Q = W = nRT \ln\left(\frac{V_2}{V_1}\right)$	$\Delta U = 0$	$W = nRT \ln\left(\frac{V_2}{V_1}\right)$
Adiabatik $Q = 0$ $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ $P_1V_1^{\gamma} = P_2V_2^{\gamma}$	$Q = 0$	$\Delta U = n\, c_v\, \Delta T$	$W = -\Delta U = -n\, c_v\, \Delta T$ $W = \frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$

Mesin Carnot



$$W = Q_1 - Q_2$$

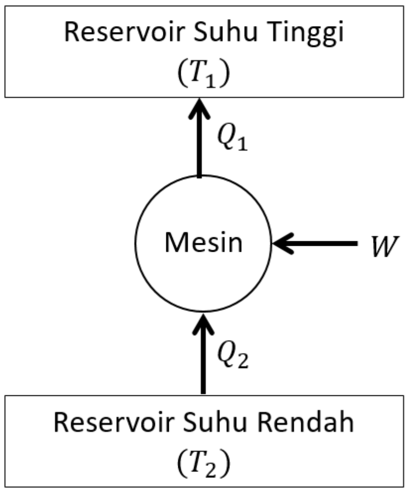
Efisiensi

$$\eta = \frac{W}{Q_1} \times 100\% = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\%$$

Efisiensi siklus Carnot

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\% = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

Mesin Pendingin



$W = Q_1 - Q_2$

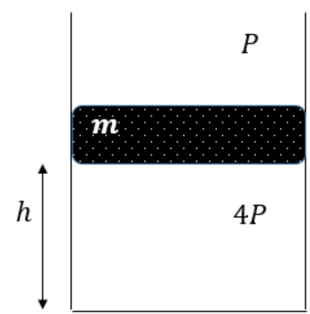
Angka Kerja

$AK = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$

Angka Kerja pada siklus Carnot

$AK = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$

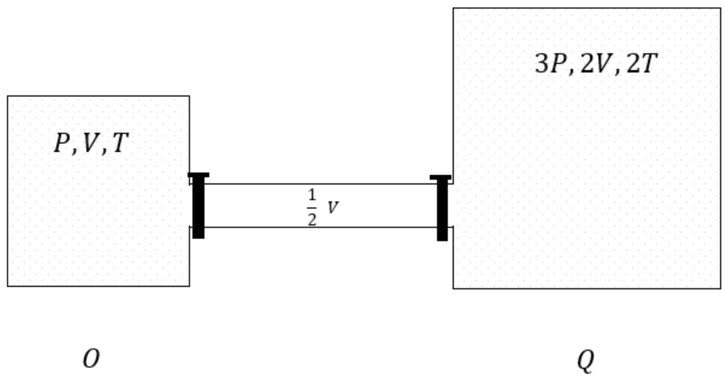
19. A vertical cylinder with a frictionless piston of mass m kg is put in the atmosphere with the gravitational acceleration of g as shown in the figure below. If two moles of a monoatomic gas is stored in the cylinder, the initial height of the gas in the cylinder is h m, and the pressure is 4 times the atmospheric pressure. Given that the cross-sectional area of the piston is A, the atmospheric pressure is P, the gas is assumed to be an ideal gas with the universal gas constant of R, and also the piston and the cylinder do not conduct heat. Answer the following questions.



a. Express mass m of the piston in terms of other quantities.
b. Express the initial temperature of the gas in terms of other quantities
c. The system in the gas is cooled down, so that the height of the gas decreased from h to $\frac{1}{2}h$. Express the magnitude of the work done on the gas in this process.
d. In the cooling process stated at c., indicate the amount of heat released by the gaseous system.

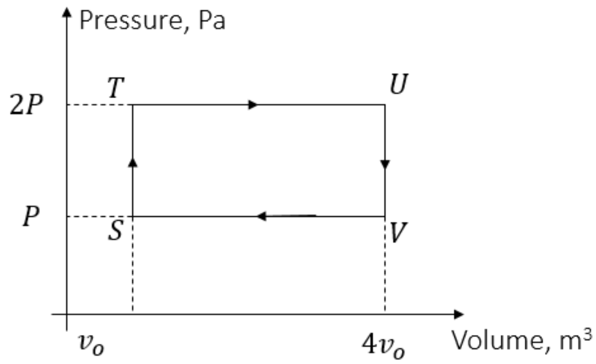
20. A monoatomic ideal gas expands from $200\ cm^3$ to $300\ cm^3$ in isobaric process with the pressure of $1 \times 10^5\ Pa$. Find the change in the internal energy of the gas.

21. Two containers O and Q with the volume of V and $2V$, respectively, are connected by a thin tube with volume of $\frac{1}{2}V$. Two valves are installed at each end of the the thin tube, and is closed initially with no gas contained in the thin tube. A monoatomic ideal gas of a pressure P and a temperature T is stored in the container O, while another monoatomic ideal gas of a pressure $3P$ and a temperature $2T$ is stored in the container Q. By assuming that the heat transfer only occurs between the gases, no heat lost to the surrounding, no reaction takes place between the gases in container O and container Q, and the universal gas constant is denoted as R, answer the following questions.



a. Express the number of moles of the gas stored in container Q initially.
b. What multiple of the number of moles of the gas stored in the container O is there in container Q?
c. What multiple of the internal energy of the gas in the container Q is there in container O?
d. After opening both valves simultaneously, the gas in container O and Q eventually reaches equilibrium. Find the temperature of the gas in equilibrium.
e. After opening both valves simultaneously, the gas in container O and Q eventually reaches equilibrium. Find the pressure of the gas.

22. One mole of a monoatomic ideal gas undergoes several stages of thermodynamics processes, starting from $S \rightarrow T \rightarrow U \rightarrow V \rightarrow S$ as shown in the figure below. Answer the following questions.



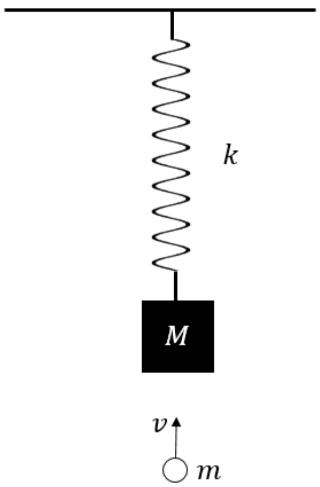
a. What multiple of temperature at U is that at S?
b. Find the expression for the work done by the gas in the process $T \rightarrow U$.
c. Among 4 stages of processes, $S \rightarrow T, T \rightarrow U, U \rightarrow V$, and $V \rightarrow S$, in which stage the gas gains the greatest thermal energy?
d. Find the thermal heat the gas receives from outside in the process of question c. Express your answer in terms of P and v_o.
e. Find the net thermal energy which the gas gains in the entire process from S to S through $S \rightarrow T \rightarrow U \rightarrow V \rightarrow S$.

23. A container holds a mixture of Oxygen gas and Argon gas. The mass ratio of Oxygen and Argon in the container is 12 : 5. Given that the relative atomic mass of Oxygen is 16 and Argon is 40. Answer the following questions.
a. The volumetric fraction of Oxygen in the mixture is approximately [...]
b. Find the total mass of the gas in the mixture if the total mole number of Oxygen and Argon gas is unity. [...]
c. If the amount of mixture stated in b. is stored at $273.15\ K$ and 0.1013×10^6 Pa, the volume of the container is approximately [...]
d. The density of the mixture in the conditions stated in c., is [...] $\frac{kg}{m^3}$
e. The ratio of the specific heat at a constant pressure to that at a constant volume of Oxygen is [...] : [...]
f. The ratio of the specific heat at a constant pressure to that at a constant volume of Argon is [...] : [...]

24. If $\log_2 6 + \log_4 x = 3$, then $x = [\ldots]$
25. If the equation $\log_2(px)\log_2(qx) + 1 = 0$ with $p > 0, q > 0$, p and q are constants and the equation has a solution $x > 0$, it follows that $\frac{p}{q} \geq [\ldots]$ or $[\ldots] < \frac{p}{q} \leq [\ldots]$.
26. If $\alpha = \log_3(2x - \sqrt{4x^2 + 1})$, then $3^{\alpha} - 3^{-\alpha} = [\ldots]x$.
27. The function $g(x) = \log_2(\log_2(\log_2(\log_2(\log_2 x))))$ has the interval $x > [\ldots]$ as its maximum domain on real numbers.
28. Given that k is a real constant. If the coefficient of x^5 term of $\left(x^2 + \frac{k}{x^3}\right)^5$ is -60, then value of $k = [\ldots]$
29. The set of solutions to the following inequality $\log_2 x + \log_2(x-1) < 3\log_8 6$ is $[\ldots] < x < [\ldots]$.
30. Given that $7^x = 5^y = 2$. If $\frac{2}{x} + \frac{1}{y}$ can be expressed as $\log_2 \alpha$, the value of $\alpha = [\ldots]$
31. If $p = \frac{\log_7 2}{\log_7 9}$, then $3^{p+1} = [\ldots]$
32. If $\frac{3^x - 3^{1-x}}{3^x + 3^{1-x}} = \frac{1}{2}$, then $x = [\ldots]$
33. The solution of the inequality $\log_{0.5}(x+2) \geq 1$ is $[\ldots] < x \leq [\ldots]$

34. Given that r is a positive constant that indicates the radius of a solid cylinder $T: x^2 + y^2 \leq r^2$ and also L is the part of cylinder that satisfies $0 \leq z \leq -y$.
    a. Let C be the plane that is formed by the intersection between L and the plane $x = a$ for $-r \leq a \leq r$, express the area of C in terms of r and a.
    b. Calculate the volume of L, and express it in terms of r.
    c. Let p be the length of the arc of the base circle of L from the point $(-r, 0,0)$ to the point $(r\cos\theta, r\sin\theta, 0)$ for which $\pi \leq \theta \leq 2\pi$. Let q be the length of the line segment from the point $(r\cos\theta, r\sin\theta, 0)$ to the point $(r\cos\theta, r\sin\theta, -r\sin\theta)$. Express p and q in terms of r and θ.
    d. Calculate the area of the side of L that intersects with $x^2 + y^2 = r^2$, and express it in terms of r.
35. Given that D is the solid shape that is formed by $0 \leq z \leq y + 4$ and $x^2 + y^2 \leq r^2$ where $0 < r \leq 4$
    a. Express the volume of D in terms of r and π
    b. Express the area of the side of D that intersects with $x^2 + y^2 = r^2$ in terms of r and π
36. Find the volume of solid shape that is formed by $z = x^2 + y^2$ and $z = 6$.
37. Given that a pyramid in the first octant bounded by $2x + 3y + z = 12$, $x = 0, y = 0$, and $z = 0$. The volume of the pyramid is [...].
38. Find the volume of solid shape that is bounded by $z = \sqrt{4 - y^2}$, $z = 0, x = 0$, and $x = 4$.

39. A light spring with negligible mass having a spring constant k is being suspended on a ceiling. An object of mass M is attached to the spring causing the spring stretched by Δx until the object reached equilibrium (at rest). A small ball with a mass of m moves vertically upward and collides with the object. Both object and small ball coalesce on impact, and then performing simple harmonic oscillation. Given that the speed of the small ball just before the collision is denoted as v and the gravitational acceleration is g, answer the following questions.



a. Construct the expression of Δx in terms of M, g, and k
b. Find the speed of the object M and the small ball m just after the collision.
c. Find the amount of decrease of the sum of kinetic energies of the small ball and the object, before and after the small ball collides with the object.
d. Find the period of the simple harmonic oscillation after the small ball and the object coalesce
e. During the simple harmonic oscillation of the small ball and the object that are stuck together, the spring is at its natural length when the object is at its highest position. Find the kinetic energy of the small ball just before it collides with the object.

40. Given that two identical springs with negligible masses and natural length of 60 cm are arranged as shown in Figure 1. Both spring are joined and their lower and upper parts are fixed in place. When small object with mass $M = 300$ g is attached in between the springs, the upper spring is extended by 10 cm and the lower spring is compressed by 10 cm, and the small object comes to rest.

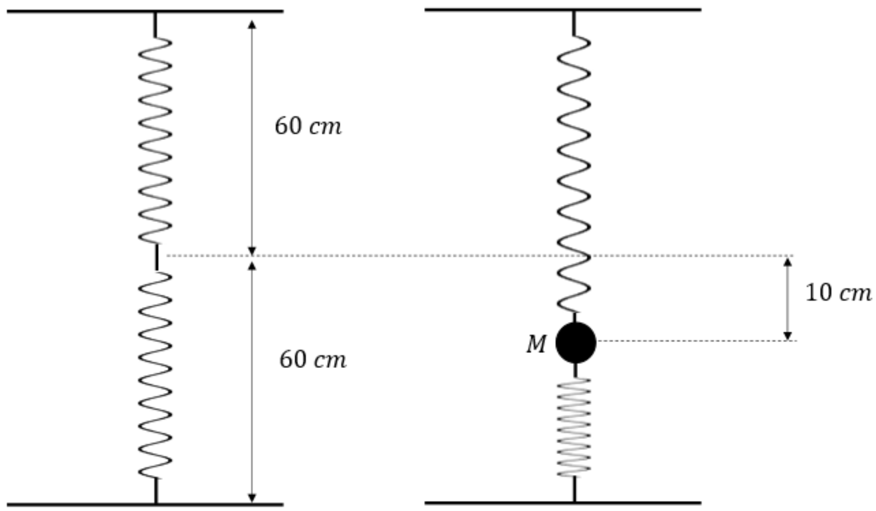


Figure 1 Figure 2

The small object is pulled upward and released gently from rest. How much time required by the small object to move from its initial launch to its lowest position? Given that gravitational acceleration is 9.8 ms^{-2}

41. A thin lightweight rubber string with the natural length of 50 cm as shown in Figure 1 is suspended on a ceiling. When a small object of mass 400 g is attached on the free end of the rubber string, the length of rubber string is extended and it becomes 65 cm as shown in Figure 2.
If the small object is pulled downward for about 30 cm from its position in Figure 2, and then gently released, what is the maximum height can be reached by the small object, measured from the launch position in Figure 3? Given that string's restoring force is proportional to the string's extension from its natural length, but the string's restoring force is not exerted when the string is not extended from its natural length.

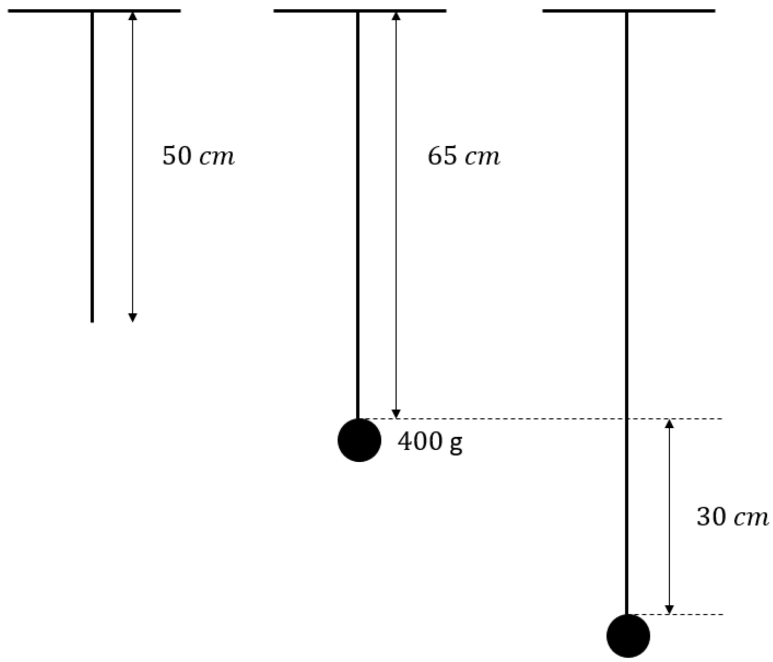


Figure 1 Figure 2 Figure 3

42. At both end of a light spring with the spring constant k, two objects with mass m_1 and m_2 are attached. Both masses and spring are placed on a flat and smooth horizontal plane (frictions on any objects and spring are negligible). Initially in figure below, the objects are suspended at rest when the spring is stretched by l. When both objects are released from rest, express the maximum velocity of the object with the mass m_1 in terms of k, l, m_1, and m_2.

Summation formula

$$\sum_{i=1}^{n} c = c + c + c + \cdots + c = cn$$

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

43. Given that d is the common difference of an arithmetic sequence denoted as $\{u_n\}$ where $n = 1,2,3, \dots$ which satisfies the three conditions as the followings.
    $u_2u_6 - u_3u_4 = -36,$ $u_5 = 24,$ $d > 0$
    a. Find the value of d
    b. Find the value of u_1 according to the value of d in a.
    c. Construct the expression of $u_n = [\dots]n - [\dots]$
    d. The sum of the first n terms is 126. Find the value of n.
44. Sequence $\{u_n\}$ is defined by $u_{n+1} - u_n = 5n,$ $u_1 = 3$ where n is a positive integer. The general term of u_n can be express as $u_n = [\dots]$
45. Given that a sequence $\{a_n\}$ satisfies that $a_1 = 1, a_2 = 2$, and $a_n - 5a_{n-1} + 4a_{n-2} = 0$ $(n \geq 3)$. The expression of $a_n = \frac{[\dots]+[\dots]}{12}$ where $n \geq 1$.
46. Sequence $\{u_n\}$ is defined by $u_{n+1} - 2u_n = -3n,$ $u_1 = 0$ where n is a positive integer. The general term of u_n can be expressed as $u_n = [\dots]$
47. Let $\{a_n\}$ be the sequence defined by
    $$a_n = \left[\frac{2n^2 + 8n + 2}{n + 5}\right]$$
    Where $[x]$ means the greatest integer that is not greater than x. Find the value of
    $$\sum_{n=1}^{30} a_n$$
48. Let b be a real number such that $3 < b < 4$. $\{b_n\}$ is the sequence defined by
    $b_1 = b, b_{n+1} = |b_n| - 3$ $(n = 1,2,3, \dots)$
    And put $Sn = b_1 + b_2 + b_3 + \cdots + b_n$
    a. Find b_4, b_5, b_6, b_7
    b. Find S_2, S_4, S_6
    c. When $n = 2m,$ where m is an integer ≥ 1, express S_n in terms of a and m
    d. When $n = 2m + 1$, where m is an integer ≥ 1, express S_n in terms of a and m